Closing Tue: 12.5(2)(3), 12.6
Closing Thu: 10.1/13.1
Office Hours: 1:30-3:00pm in Com B-006

12.5 Summary

Lines: $x = x_0 + at$, $y = y_0 + bt$, $z = z_0 + ct$.

$\boldsymbol{v} = \langle a, b, c \rangle$ = a direction vector

$\boldsymbol{r_0} = \langle x_0, y_0, z_0 \rangle$ = a position vector

To find equations for a line

- Info given? → Find two points
- Done. → $\vec{\boldsymbol{v}} = \overrightarrow{AB}$ (subtract components), $\overrightarrow{\boldsymbol{r_0}} = \vec{A}$

lines parallel – direction vectors parallel.
lines intersect – possible to make (x,y,z) all equal (diff. param.)
Otherwise, we say they are skew.

Planes:

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

$\boldsymbol{n} = \langle a, b, c \rangle$ = a **normal** vector.

$\boldsymbol{r_0} = \langle x_0, y_0, z_0 \rangle$ = a position vector

To find the equation for a plane

- Info given? → Find three points
- Done. → Two vectors parallel to the plane: $\overrightarrow{AB}$ and $\overrightarrow{AC}$
- $\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC}$, $\overrightarrow{\boldsymbol{r_0}} = \vec{A}$

planes parallel – normal vectors parallel.
Otherwise, the planes intersect.
Acute angle of intersection is the acute angle between their normal vectors.

1. Find an equation for the line that goes through the two points $A(1,0,-2)$ and $B(4,-2,3)$.
2. Find an equation for the line that is parallel to the line $x = 3 - t$, $y = 6t, z = 7t + 2$ and goes through the point $P(0,1,2)$.
3. Find an equation for the line that is orthogonal to $3x - y + 2z = 10$ and goes through the point $P(1,4,-2)$.
4. Find an equation for the line of intersection of the planes
   $5x + y + z = 4$ and
   $10x + y - z = 6$.

1. Find the equation of the plane that goes through the three points A(0,3,4), B(1,2,0), and C(-1,6,4).

2. Find the equation of the plane that is orthogonal to the line $x = 4 + t, y = 1 - 2t, z = 8t$ and goes through the point P(3,2,1).

3. Find the equation of the plane that is parallel to $5x - 3y + 2z = 6$ and goes through the point P(4,-1,2).

4. Find the equation of the plane that contains the intersecting lines

$x = 4 + t_1, y = 2t_1, z = 1 - 3t_1$

and

$x = 4 - 3t_2, y = 3t_2, z = 1 + 2t_2.$

5. Find the equation of the plane that is orthogonal to $3x + 2y - z = 4$ and goes through the points P(1,2,4) and Q(-1,3,2).

Questions directly from old tests:

1. Consider the line thru (0, 3, 5) that is orthogonal to the plane $2x - y + z = 20.$ Find the point of intersection of the line and the plane.

2. Find the equation for the plane that contains the line $x = t, y = 1 - 2t, z = 4$ and the point (3,-1,5).

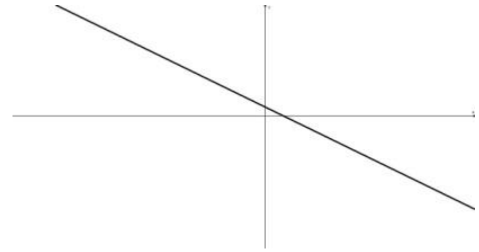
12.6: A few basic 3D surfaces

First, a 2D review.

Examples:

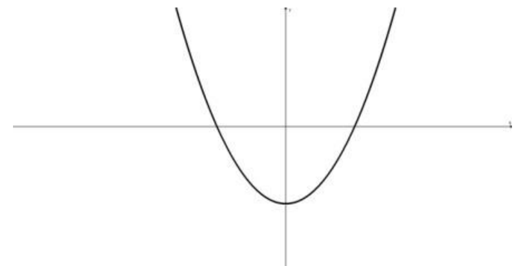
Line: $ax + by = c$

$3x + 2y = 1$



Parabola: $ax^2 + by = c$ or
$ax + by^2 = c$

$3x^2 - y = 4$

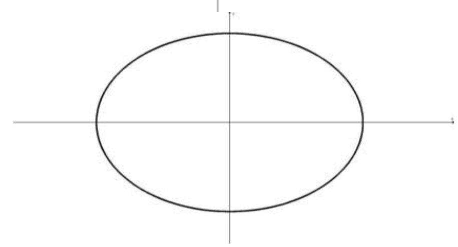


Ellipse: $ax^2 + by^2 = c$ (if $a,b,c > 0$)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

(*Note:* If a = b, then it's a circle)

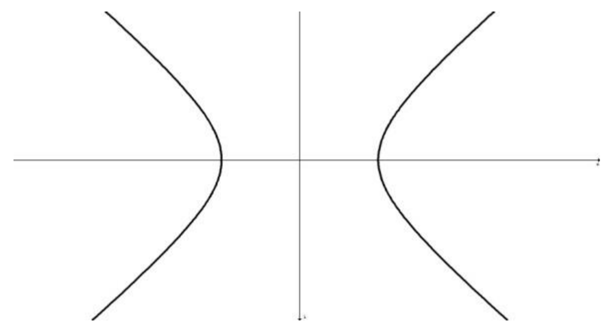
$$\frac{x^2}{3^2} + \frac{y^2}{2^2} = 1$$



Hyperbola: $ax^2 - by^2 = c$ or
$-ax^2 + by^2 = c$ (if $a,b,c > 0$)

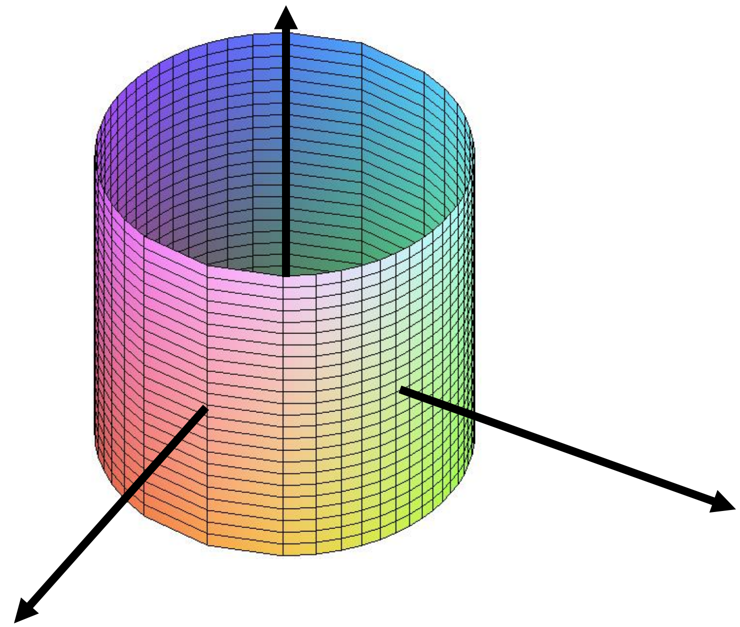
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{3^2} - \frac{y^2}{4^2} = 1$$



12.6: A few basic 3D Shapes

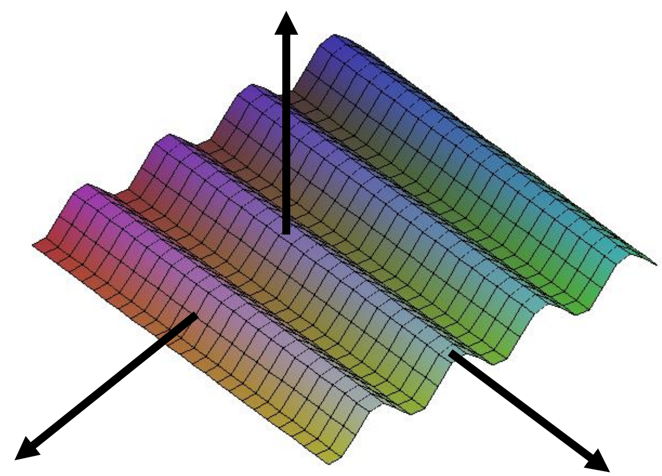
Cylinders: If *one variable is absent*, then the graph is a 2D curve extended into 3D.



If the 2D shade is called "BLAH", then the 3D shade is called a "BLAH cylinder".

Examples:

(a) $x^2 + y^2 = 1$ in 3D is a **circular cylinder** (*i.e.* a circle extended in the *z*-axis direction).

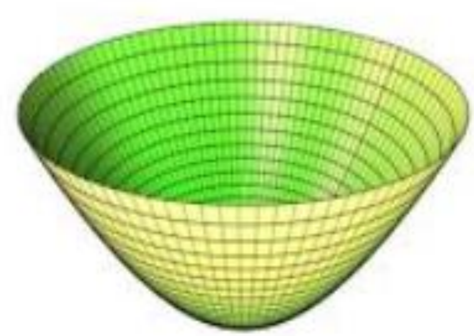


(b) z = cos(x) in 3D is a **cosine cylinder** (i.e. the cosine function extended in the *y*-axis direction).

Quadric Surfaces: A surface given by an equation involving a sum of first and second powers of *x*, *y*, and *z* is called a *quadric surface*.

To visualize, we use **traces**:
Fix one variable and look at the resulting 2D picture (i.e. look at one slice).

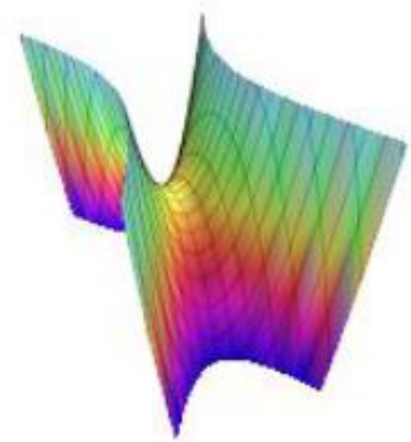
If we do several traces in different directions, we start to get an idea about the picture.



Elliptical/Circular Paraboloid

$$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

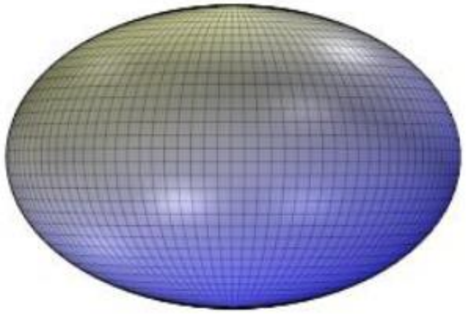
(*ex*: $z = 3x^2 + 5y^2$)



Hyperbolic Paraboloid

$$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

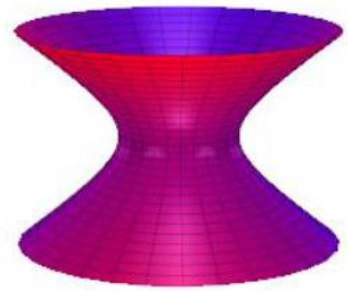
(*ex*: $y = 2x^2 - 5z^2$)



Ellipsoid/Sphere

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

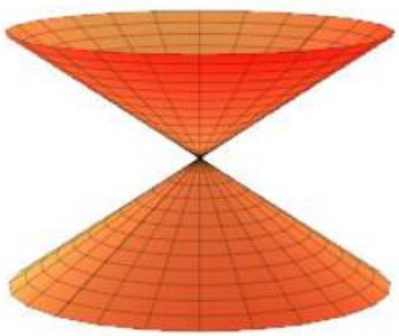
(*ex*: $3x^2 + 5y^2 + z^2 = 3$)



Hyperboloid of One Sheet

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$

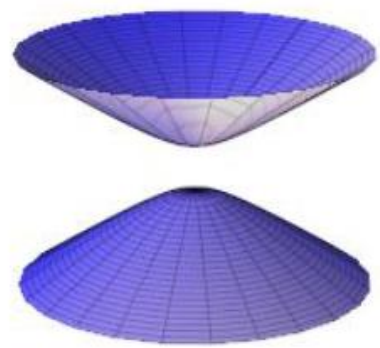
(*ex*: $x^2 - y^2 + z^2 = 10$)



Circular/Elliptical Cone

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$$

(*ex*: $z^2 = x^2 + y^2$)



Hyperboloid of Two Sheets

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$$

(*ex*: $x^2 + y^2 - z^2 = -4$)

Practice Examples

Find the traces and name the shapes:

1. $x - 3y^2 + 2z^2 = 0$
2. $4x^2+3y^2 = 10$
3. $5x^2 - y^2 - z^2 = 4$
4. $-x^2 + y^2 + 4z^2 = 0$
5. $x^2 - 2y^2 + z^2 - 6 = 0$

Answers:

1. $x - 3y^2 + 2z^2 = 0$

   $x = k$: $k - 3y^2 + 2z^2 = 0$ (**hyp.**)
   $y = k$: $x - 3k^2 + 2z^2 = 0$ (**par.**)
   $z = k$: $x - 3y^2 + 2k^2 = 0$ (**par.**)
   Also note: $x = 3y^2 - 2z^2$

   Name: **Hyperbolic paraboloid**

2. $4x^2+3y^2 = 10$

   One variable missing.
   The given equation is an ellipse in the xy-plane.

   Name: **Elliptical Cylinder**

3. $5x^2 - y^2 - z^2 = 4$

   $x = k$: $5k^2 - y^2 - z^2 = 4$ (circ/pt/nothing)
   $y = k$: $5x^2 - k^2 - z^2 = 4$ (**hyp**)
   $z = k$: $5x^2 - y^2 - k^2 = 4$ (**hyp**)
   Also note: $-5x^2 + y^2 + z^2 = -4$

   Name: **Hyperboloid of Two Sheets**

4. $-x^2 + y^2 + 4z^2 = 0$

$x = k$: $-k^2 + y^2 + 4z^2 = 0$ (**ellipse/pt**)
$y = k$: $-x^2 + k^2 + 4z^2 = 0$ (**hyp./lines**)
$z = k$: $-x^2 + y^2 + 4k^2 = 0$ (**hyp./lines**)

Also note: $x^2 = y^2 + 4z^2$

Name: **Elliptical Cone**

5. $x^2 - 2y^2 + z^2 - 6 = 0$

$x = k$: $k^2 - 2y^2 + z^2 - 6 = 0$ (**hyp**)
$y = k$: $x^2 - 2k^2 + z^2 - 6 = 0$ (**circle**)
$z = k$: $x^2 - 2y^2 + k^2 - 6 = 0$ (**hyp**)

Also note: $x^2 - 2y^2 + z^2 = 6$

Name: **Hyperboloid of One Sheet**